N7 • Using percentages to increase quantities

Mathematical goals

To enable learners to:

- make links between percentages, decimals and fractions;
- represent percentage increase and decrease as multiplication;
- recognise the inverse relationship between increases and decreases.

Starting points

Most learners will have met these concepts before. Many, however, will have been introduced to the ideas in a procedural manner ("this is how you calculate a percentage of a quantity") rather than in a conceptual one. Gaps and misconceptions will remain. Typically, for example, we find that learners believe an increase of 50% followed by a decrease in 50% takes us back to the original value. This session will confront such misconceptions and build new conceptual links between percentages, decimals and fractions.

Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Money*;
- Card set B – *Percentages*;
- Card set C – *Words*;
- Card set D – *Decimals*;
- Card set E – *Fractions*;
- calculators.

Time needed

From 1 to 2 hours, depending on how many card sets are used.

Suggested approach

Beginning the session

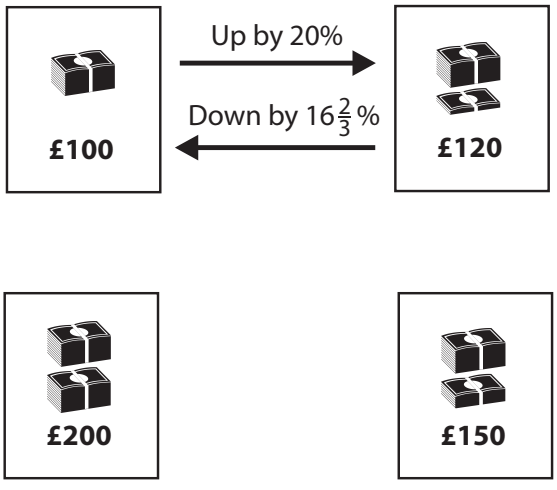
Ask learners to tackle the following question on their own, without discussion.

> In a sale, the prices in a shop were all reduced by 33%.
> After the sale they were all increased by 50%.
> What was the overall effect on the shop prices?
> Explain how you know.

Experience suggests that most learners will answer that prices have risen by 17%. The intention here is simply to expose existing thinking. Towards the end of the session, learners will reconsider their answers and put them right.

Working in groups

Ask learners to work in pairs or threes. They will need plenty of table space for the activity, so you may need to push tables together. Give each group Card sets A – *Money* and B – *Percentages*. Ask learners to place the money cards in the form of a square on the table, so that their values increase in a clockwise direction, starting at top left, as shown in the diagram.



Learners must take it in turns to place pairs of *Percentages* cards between each pair of *Money* cards to show the correct percentage increase or decrease. Pairs may be horizontal, vertical or diagonal. Blank cards are provided if learners wish to make new cards.

If you think learners will struggle with this, you may like to start with just three *Money* cards (£100, £150, £200) and the corresponding *Words* (Card set C), *Decimals* (Card set D) and/or *Fractions* (Card set E) cards. If you think that some learners may find the work straightforward, you may like to make the *Money* cards more complex (e.g. 80p, 96p, £1.60, £1.20).

Typically, learners make the mistake of pairing an increase of 50%

with a decrease of 50%, and so on. It is important that you do not comment on this at this stage. You should wait to see if learners can correct these mistakes for themselves, as more cards are added.

When learners have done this, introduce Card sets C and D. Ask learners to add these to the arrangement on the table. These cards provide learners with different ways of interpreting the situation. Allow access to calculators to facilitate the arithmetic.

Finally, when learners have completed this stage, give out Card set E and ask learners to place these in position.

As you monitor the work of the groups, listen to their discussions. Help learners to look for patterns and generalisations in their results, perhaps by highlighting some yourself. Learners may notice the following.

- An increase of, say, 33% is equivalent to multiplying by 1.33. So, an easy way to increase by n% is to write n% as a decimal, add one and multiply by this number.
- A decrease of, say, 33% is equivalent to multiplying by (1–0.33) = 0.67. So, an easy way of decreasing by n% is to write n% as a decimal, subtract it from one and multiply by the resulting number.
- The inverse of an increase by a percentage is not a decrease by the same percentage.
- There is a clear pattern in the pairs of words that represent inverse functions:

Doubled	Down by one half
Up by one half	Down by one third
Up by one third	Down by one quarter
Up by one quarter	Down by one fifth
Up by one fifth	Down by one sixth

- This pattern is less clear when we consider decimals, although a calculator enables us to easily show that each pair multiplies to give 1.

×2	×0.5	and $2 \times 0.5 = 1$
×1.5	×0.$\dot{6}$	and $1.5 \times 0.\dot{6} = 1$
×1.$\dot{3}$	×0.75	and $1.\dot{3} \times 0.75 = 1$
×1.25	×0.8	and $1.25 \times 0.8 = 1$
×1.2	×0.8$\dot{3}$	and $1.2 \times 0.8\dot{3} = 1$

- The pattern in the pairs of fraction multipliers is very easy to see:

$\times \frac{2}{1}$ $\times \frac{1}{2}$

$\times \frac{3}{2}$ $\times \frac{2}{3}$

$\times \frac{4}{3}$ $\times \frac{3}{4}$

. . . and so on.

Reviewing and extending learning

Discuss and generalise what has been learned with the whole group, using mini-whiteboards and questions pitched at appropriate levels.

If a price increases by 10% . . .

- How can you write that in words?
- How can you write that as a decimal multiplication?
- How can you write that as a fraction multiplication?

How much will the price need to go down to get back to the original price?

- How can you write that in words?
- How can you write that as a decimal multiplication?
- How can you write that as a fraction multiplication?

Finally, reconsider the problem that began this session. How would learners now answer this differently?

What learners might do next

If you want to continue the session on another day, then begin with a different, harder, set of *Money* cards. Alternatively, you could use shapes and discuss enlargement by different scale factors.

You could also ask learners to create their own sets of cards using the generalisations they have deduced in this session.

Further ideas

This session uses multiple representations of states and transformations. Similar activities in other mathematical contexts are included in this pack. For example:

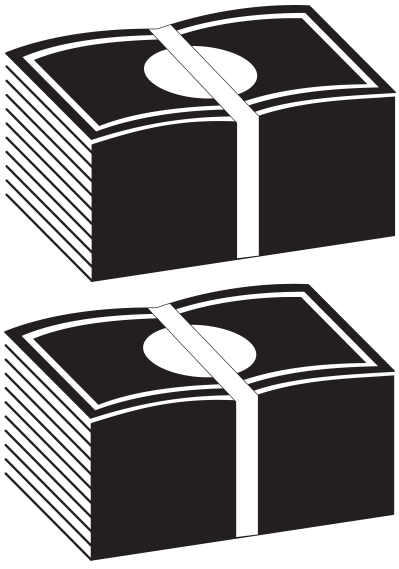
N8 Using directed numbers in context
(states are temperatures; transformations are rises and falls);

SS7 Transforming shapes
(states are shapes; transformations are translations, reflections and rotations).

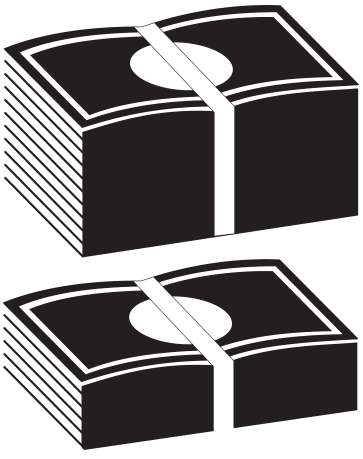
N7 Card set A – *Money*

£100

£120



£200

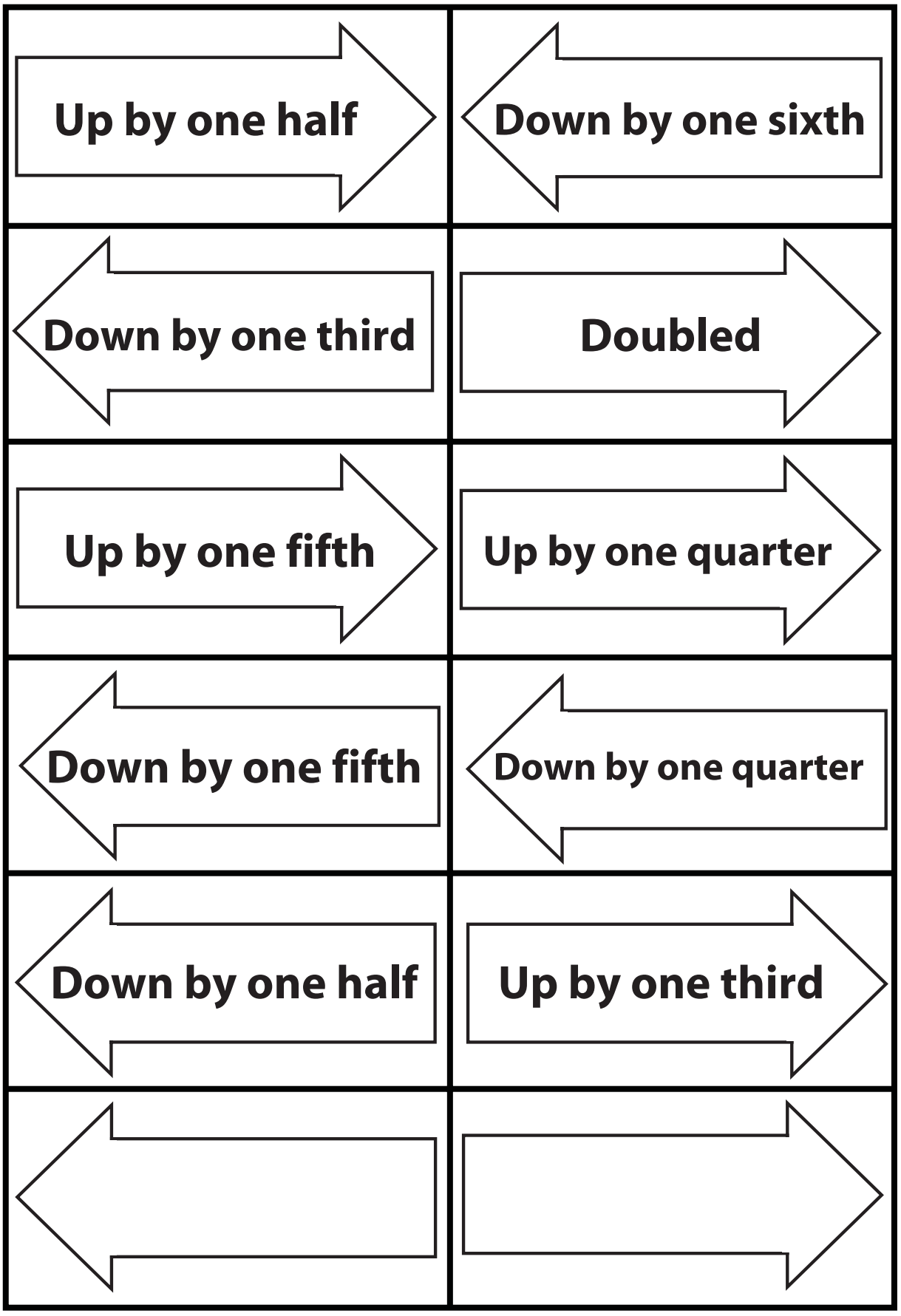


£150

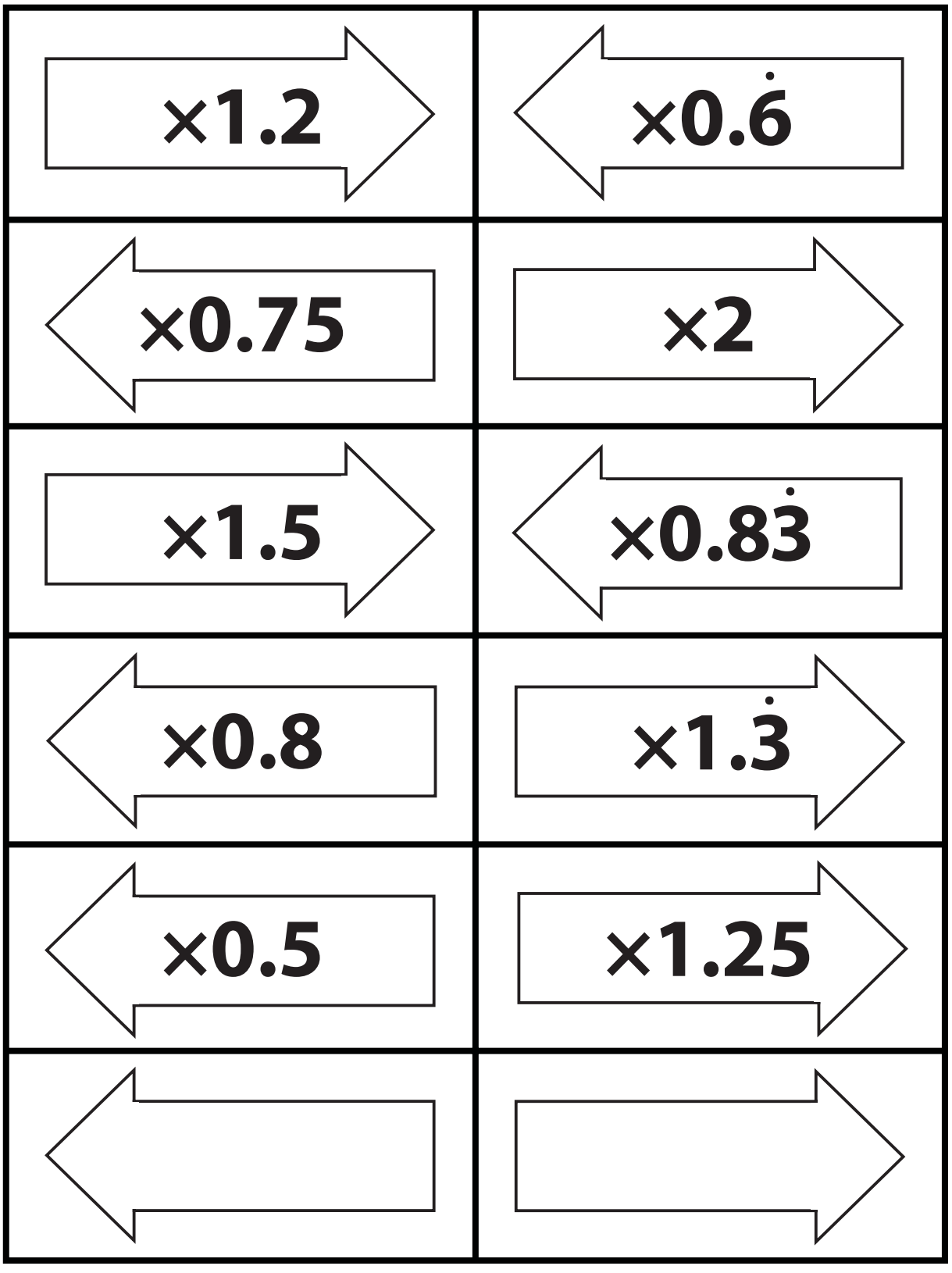
N7 Card set B – *Percentages*

Down by 50%	Down by 20%
Up by 25%	Up by 20%
Down by $33\frac{1}{3}\%$	Down by $16\frac{2}{3}\%$
Down by 25%	Up by 50%
Up by $33\frac{1}{3}\%$	Up by 100%

N7 Card set C – *Words*



N7 Card set D – *Decimals*



N7 Card set E – *Fractions*